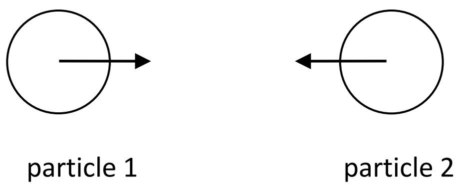
Teacher notes

Topic D

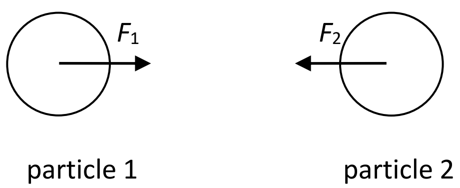
Negative mass?

It is tempting to consider the theoretical possibility of negative mass! What kind of Physics would it lead to?

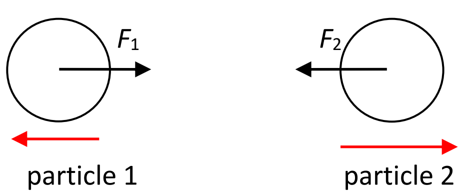
We can examine this possibility by assuming that Newton's second law of mechanics and Newton's law of gravitation stay the same. I.e. $\vec{F} = m\vec{a}$ and $\vec{F} = \frac{Gm_1m_2}{r^2}\hat{r}$. In the second formula $\hat{r}$ is a unit vector in the direction from particle 1 to particle 2. This gives the gravitational force on particle 1. The gravitational force on particle 2 is $-\vec{F}$. The situation shown in the figure applies to two particles of positive mass.



How do things change if both particles have negative masses $-m_1$ and $-m_2$? The gravitational force on particle 1 is $\vec{F}_1 = \frac{G(-m_1)(-m_2)}{r^2}\hat{r} = \frac{Gm_1m_2}{r^2}\hat{r}$ and on particle 2 it is $\vec{F}_2 = -\frac{G(-m_1)(-m_2)}{r^2}\hat{r} = -\frac{Gm_1m_2}{r^2}\hat{r}$.



But now, $(-m_1)\vec{a}_1 = \frac{Gm_1m_2}{r^2}\hat{r} \Rightarrow \vec{a}_1 = -\frac{Gm_2}{r^2}\hat{r}$ and $(-m_2)\vec{a}_2 = -\frac{Gm_1m_2}{r^2}\hat{r} \Rightarrow \vec{a}_2 = +\frac{Gm_1}{r^2}\hat{r}$. (Accelerations shown in red arrows.)

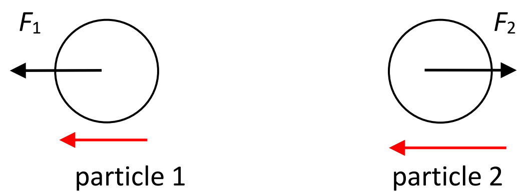


The accelerations are opposite to each other so the two particles of negative mass actually repel!

To make matters worse consider the case of one positive and one negative mass (m_1 and $-m_2$). The forces are

$$\vec{F}_1 = \frac{G(m_1)(-m_2)}{r^2}\hat{r} = -\frac{Gm_1m_2}{r^2}\hat{r} \text{ and } \vec{F}_2 = -\frac{G(m_1)(-m_2)}{r^2}\hat{r} = +\frac{Gm_1m_2}{r^2}\hat{r}$$

And the accelerations are as shown by the red arrows.



The particles move in the same direction! This is contrary to science fiction stories where positive and negative masses repel, a kind of anti-gravity.

Initially the particles are at rest and so the total momentum is zero. Now the particles move so the total momentum is (the overall minus sign is because both move to the left)

$$P = -\left(m_1v_1 + (-m_2)v_2\right)$$

If the masses are equal in magnitude then

$$P = -mv_1 + mv_2 = -m(v_1 - v_2)$$

And this remains zero since in this case $v_1 = v_2$.

However, if the masses are different $P \neq 0$ and the concept of negative mass has led to a violation of momentum conservation!